1. Sketch the graph of a function $f: \mathbb{R} \to \mathbb{R}$ satisfying the following conditions:

(a) $f$ is continuous, $f'(x) < 0$ for $x < 2$, $f'(x) > 0$ for $x > 2$, $f'(2) = -1$, $f'_+(2) = \infty$, $\lim_{x \to -\infty} f'(x) = 0$.

(b) $f$ is continuous at all points $x \in \mathbb{R} - \{1\}$, $x = 1$ is the equation of its left-side vertical asymptote, but not the right-side one, $f'(4)$ does not exist, $f'(x) < 0$ for $x < -4$, $\lim_{x \to -\infty} f(x) = 6$, a line given by $y = x - 1$ is its oblique asymptote at $+\infty$.

(c) $f$ is continuous at all points $x \in \mathbb{R} - \{-1, 2\}$, line given by $x = -1$ is its right-side vertical asymptote, but not the left-side one, a line given by $x = 2$ is its vertical asymptote, $f'(x) < 0$ for $x < 0$, at $0$ function has an inflection point, $\lim_{x \to -\infty} f(x) = 4$, $\lim_{x \to -\infty} f'(x) = 0$.

2. Show that:

(a) $e^x > 1 + x$ for $x \neq 0$,
(b) $\ln x > \frac{2(x-1)}{x+1}$ for $x > 1$,
(c) $\ln(1 + x) > \frac{\arctan x}{x+1}$ for $x > 0$,
(d) $\frac{x}{x+1} \leq \ln(1 + x) \leq x$ for $x > -1$,
(e) $2 \arctan x + \arcsin \frac{2x}{1+x^2} = \pi$ for $x \geq 1$,
(f) $2x \arctan x > \ln(1 + x^2)$ for $x > 0$,
(g) $\log_2 3 > \log_3 4$.

3. Find asymptotic lines for the following functions:

(a) $f(x) = \frac{1}{x} + x \arctan x$,
(b) $f(x) = (x + 2)e^{\frac{1}{x}}$,
(c) $f(x) = x + \frac{\ln x}{x}$,
(d) $f(x) = \sqrt{\frac{x^3}{x-1}}$.

4. Let

$$f(x) = \begin{cases} e^{-x} & \text{for } x \leq 0, \\ 4x - x^3 & \text{for } x > 0. \end{cases}$$

Find the intervals on which $f$ is decreasing, increasing, concave up, concave down. Find local extreme values and inflection points of $f$.

5. Find the intervals on which the function $f(x) = \ln(x + \sqrt{x^2 + 1})$ is both increasing and concave up.