Spectral decay of Finite Fourier Transforms and related random matrices.
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The finite Fourier transform operator, and in particular its singular values, have been extensively studied in relation with band-limited functions. Recall that, for $m$ a positive number, the finite Fourier transform $F_m$ is defined on $L^2(-1/2, +1/2)$ by

$$F_m(f)(y) = \sqrt{m} \int_{-1/2}^{+1/2} \exp(2i\pi myz)f(z)dz, \quad |y| < 1/2.$$ 

We study singular values of the $n \times n$ matrix whose $j,k$ coefficient is given by $\exp(2i\pi Y_j Z_k)$, with $Y_j, Z_k$ i.i.d and uniformly distributed on $(-1/2, +1/2)$. This may be seen as a random discretization of $F_m$. This type of matrices has been introduced by Desgroseilliers, Lévêque and Preissmann as an approximate model of a wireless communication network. We prove that, with high probability, the sequence of singular values of the random matrix is close to the sequence of singular values of the finite Fourier transform itself. As an application, we prove that the number of degrees of freedom of this approximate model is well approximated by $m$ with high probability.

Our main tool is a study of kernel matrices $\kappa(Y_j, Y_k)$, where $Y_j$ is a sample of some probability law and $\kappa$ is a positive semi-definite kernel. These matrices have been very much studied in relation with deep learning. We give $\ell^2$ estimates on the spectrum of such matrices, which strenghtens separate estimates for each eigenvalue given by J. Shawe-Taylor and N. Cristianini on one hand, G. Blanchard, O. Bousquet and L. Zwald on the other.

This is issued from a joint work with Abderrazek Karoui.