Finite distributive lattices with the antitone bijections on the principal filters

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The system \( A = (A, \vee, \wedge, (f_a)_{a \in A}, 1, 0) \) satisfying conditions

- \((A, \vee, \wedge, 0, 1)\) is a bounded lattice,
- \( f_a \) is antitone isomorphism on the interval \([a, 1]\) (for any \( a \in A \)), (more precisely, inequality \( f_a(x) \leq f_a(y) \) holds if and only if \( y \leq x \) holds for any \( x, y \in A \)).

is called to be a lattice with section antitone bijections.

Moreover, if the mappings \( f_a \) are involutions (thus \( f_a(f_a(x)) = x \) holds for any \( a \in A \) and \( x \in [a, 1] \)) then the system is called to be a lattice with section antitone involutions.

The basic algebras was introduced in [2] as algebraic representation of the lattices with section antitone involutions. We recall that the algebra \( A = (A, \oplus, \neg, 0) \) of type \( \langle 2, 1, 0 \rangle \) is a basic algebra if it satisfies the identities

\begin{align*}
\text{BA1} & \quad x \oplus 0 = x, \\
\text{BA2} & \quad \neg \neg x = x, \\
\text{BA3} & \quad \neg(\neg x \oplus y) \oplus y = \neg(\neg y \oplus x) \oplus x, \\
\text{BA4} & \quad \neg(\neg(\neg(x \oplus y) \oplus y) \oplus z) \oplus (x \oplus z) = 1.
\end{align*}

It was proved in [1] that a finite commutative basic algebras (the basic algebras with a commutative operation ‘\( \oplus \)’) are just a finite MV algebras (the operation ‘\( \oplus \)’ is also associative), see [3].

We will prove the theorem which is motivated by previous results:

**Theorem 1.** If the system \( A = (A, \vee, \wedge, (f_a)_{a \in A}, 1, 0) \) is a lattice with section antitone bijections such that \((A, \vee, \wedge)\) is distributive lattice then the lattice \((A, \vee, \wedge)\) is a direct product of the finite chains.

**References**

