Exact Motion Planning with Rotations: the Case of a Rotating Polyhedron

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Problem description

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Configuration space

The set of all configurations.

Examples of configuration space:

- $\mathbb{R}^3 x SO(3)$ - polyhedron motion in $\mathbb{R}^3$ with rotations
- $\mathbb{R}^2 x SO(2)$ - polygon motion in $\mathbb{R}^2$ with rotations

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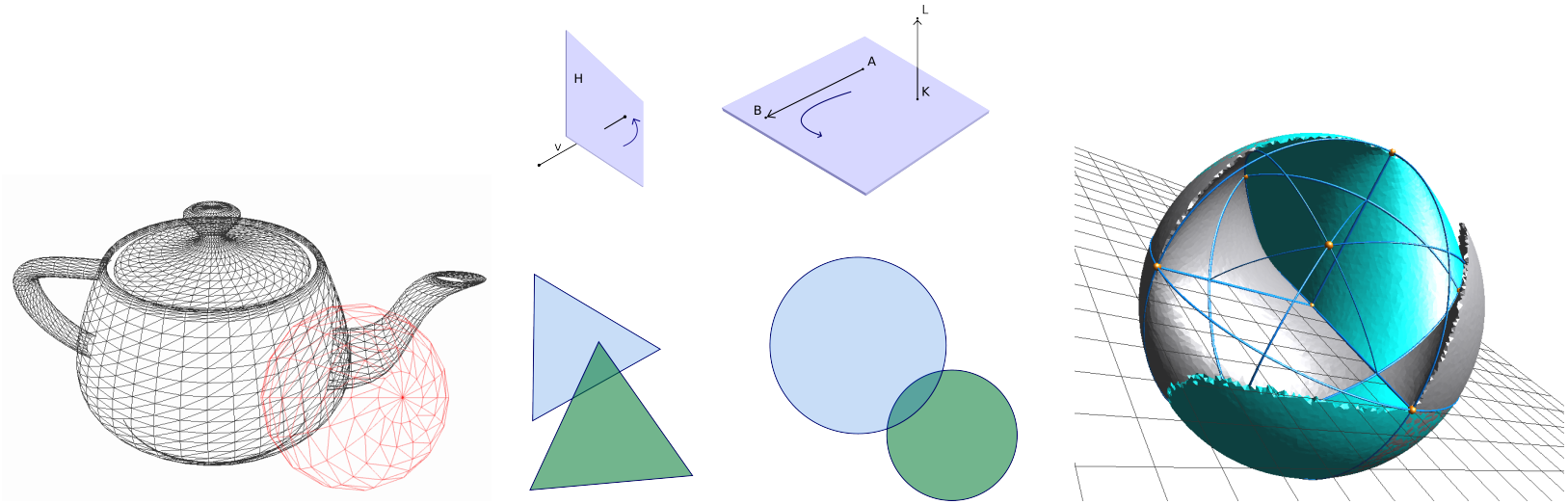
Forbidden (configuration space

The subset of a configuration space which represent rotations which cause at least one collision of the rotating polyhedron with obstacles.

libcs library: the approach for solving the problem



(a) Scene (b) Predicates (c) C-space

Figure: libcs algorithm overview

libcs library: new theoretical results

Theorem (Exact predicates)

The following predicates can be represented exactly:

- *plane-point, plane-ball collisions*
- *ball-ball collision*
- *triangle-triangle collision*

They are represented by quadrics in Spin(3). Each quadric divides the whole space into a colliding part (forbidden c-space) and non-colliding part (empty c-space).

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It is possible to implement surface intersection (and thus, a motion planning algorithm) in SO(3) by using a library for quadric intersection in $\mathbb{P}^3$.

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libqi - a complete library for quadric intersection

The algorithm

Configuration space construction algorithm:

- collect a list of predicates from scene
- calculate surfaces in configuration space
- intersect all surfaces - edges of the graph
- intersect edges of the graph with all surfaces - vertices of the graph
- collecting adjacency information and final graph construction

Result: intersection graph in configuration space in which we can locate desired motion paths.

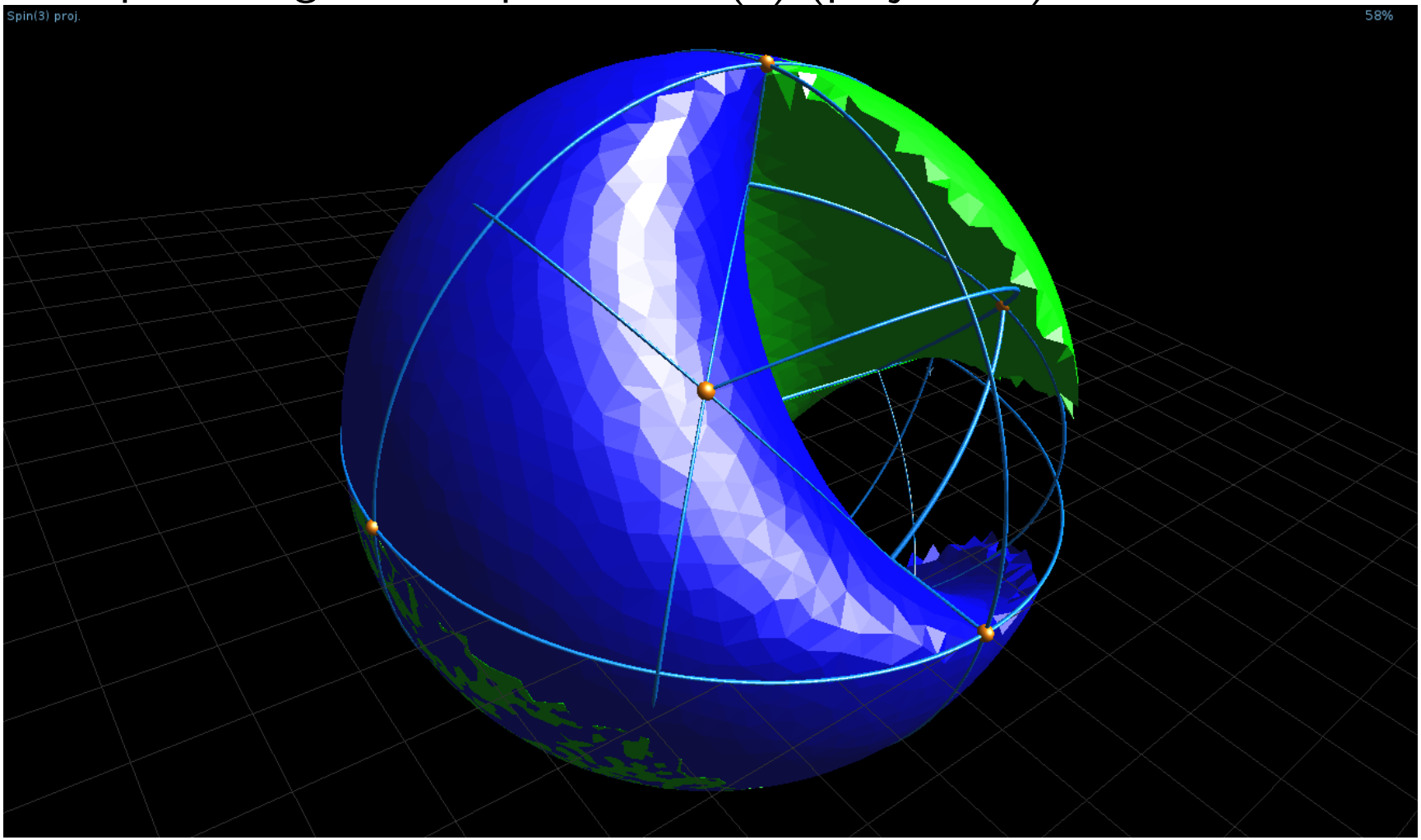
libcs library: implementation

libcs: a library for computing configuration spaces.

property	value	comments
c-space	Spin(3)	
rot. rep.	spinors	or quaternions
result rep.	graph of intersections	
code	templated C/C++	based on CGAL
arithmetic	gmp Z, double,	provided by CGAL
status	implemented	partially optimized
complexity	$O(m^3 n^3 log(mn))$	output sensitive

The result of algorithm

Example configuration space in SO(3) (projection).



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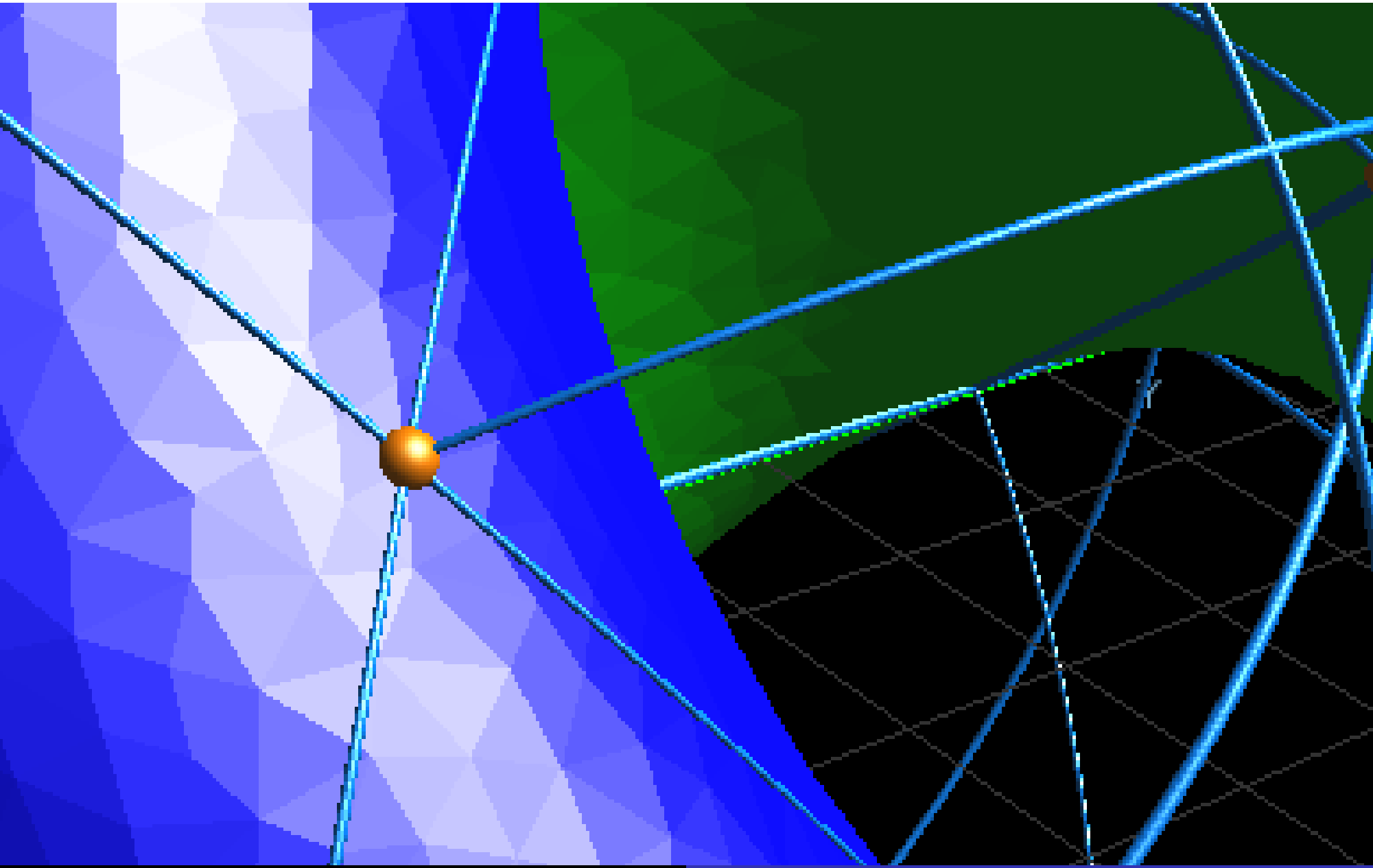

Conclusions

- First exact motion planning algorithm involving 3D rotations
- First exact representation of SO(3) configuration space
- An ingredient for other motion planning algorithms (involving 3D rotations)
- algorithms in SO(3) are very different from similar ones in $\mathbb{R}^3$ (difficult localization)
- Near-optimal algorithm and output sensitive

Thank you