An Efficient Method of Constructing Cell Graph in Arrangement of Quadrics in 3-dimensional Space of Rotations

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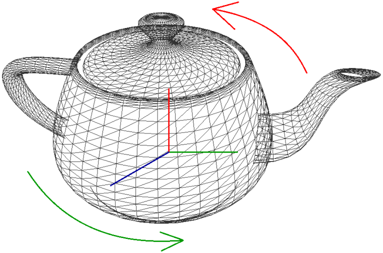
Introduction

Assume there is a scene consisting of a number of polyhedral obstacles. A selected polyhedron, which is able to rotate around some predefined rotation center, is called a *rotating polyhedron*.

A (rotational) motion planning task

A motion planning task is a problem of finding a continuous collision-free rotation path of a *rotating polyhedron* admist polyhedral obstacles from the selected initial rotation to the terminal rotation.



Configuration space approach

A motion planning problem can be reformulated in terms of a configuration space. Configurations are represented by spinors (isomorphic to quaternions). Each rotation is described by four numbers.

Theorem (Dobrowolski, 2012)

A motion planning problem in $SO(3)$ is equivalent to calculation of an arrangement of quadratic surfaces in S^4 (3-dimensional unit sphere).

Corollary:
An effective motion planning algorithm can be built by providing an efficient way of calculating an arrangement of quadrics in S^4.
A quadric in Spin(3) space is called a **spin-quadric**.

The cell algorithm

Approximate configuration space construction algorithm:

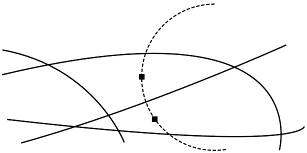
- create a list of predicates from a scene
- create a list of spin-quadrics from the predicate list
- remove all duplicates in the list of spin-quadrics
- locate a set of cells in the arrangement of spin-quadrics
- locate neighbouring cells in the set of cells
- compose a cell graph in the set of cells

Result: a cell graph in configuration space in which it is relatively easy to locate desired motion paths.

Locating cells in an arrangement

A method of choosing samples from the configuration space. A single sample defines the surrounding cell unambiguously. Two approaches for a sample generator:

- Naive sample generator
  Generates samples distributed uniformly in Spin(3)
- QC sample generator
  Choose a random 1-dimensional set of rotations (which forms a circle of rotations) and intersect it with all the spin-quadrics in an arrangement



Practical results show that each approach has different characteristics.

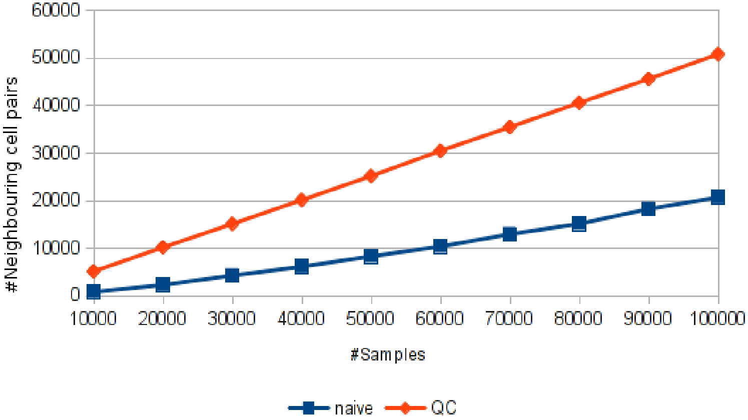
Neighbour searching algorithm

A method of locating all pairs of neighbouring cell pairs in an arrangement. Assume that there are n cell coordinates of length l. Three different approaches were proposed:

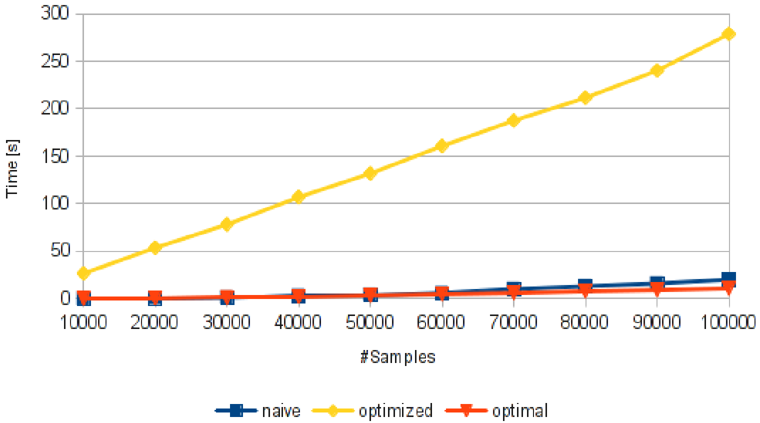
- Naive approach $O(n^2 l)$
  Check all possible pairs of coordinates. Given a pair of coordinates their neighbourhood is established in $O(l)$ time
- Optimized approach $O(nl^2)$
  Create a lookup search tree for looking up cell coordinates. For each coordinate and its bit check whether there is a neighbour coordinate with a changed bit.
- Optimal approach $O(nl)$
  Use lookup search tree in a different way - traverse it from the leafs to the root and collect neighbour pairs. An optimal solution.

Performance of the new methods

The quality of a sample generator measured as the number of induced neighbouring cell pairs.

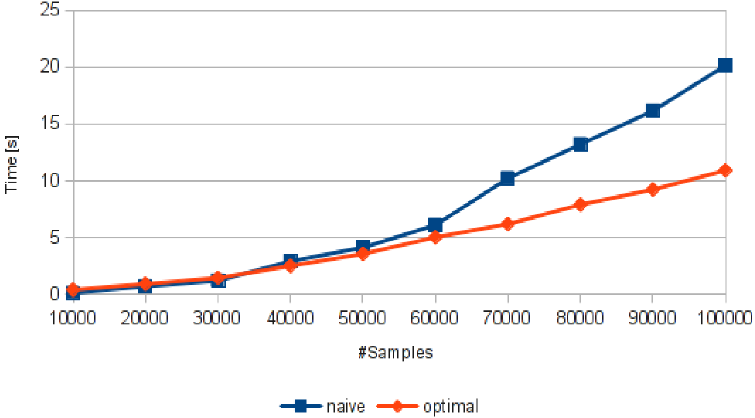


Performance of the cell neighbour searching algorithm



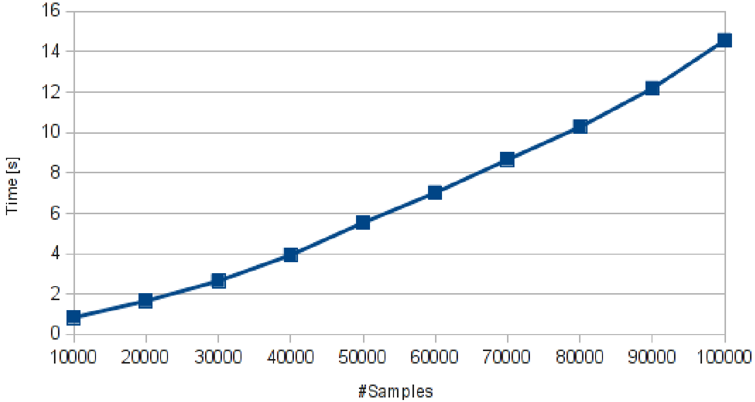
Performance of the new methods

Performance of the cell neighbour searching algorithm (II)



Performance of the new methods

Performance of the complete implementation with the QC sample generator and optimal neighbour searching algorithm



Conclusions

- A practical motion planning algorithm involving 3D rotations
- A good quality approximation of SO(3) configuration space
- An ingredient for other motion planning algorithms (involving 3D rotations)

Further reading

More on this topic:

- the exact version of rotational motion planning
  "An Algorithm for Computing the Exact Configuration Space of a Rotating Object in 3-space"
  IAENG International Journal of Computer Science, Volume 39 Issue 4, Pages 363-376
- a complete survey on rotational motion planning
  "Evaluation of the usefulness of exact methods to motion planning in configuration space"
  PhD, Warsaw University of Technology (to appear)

Thank you