1. **Equivalence relation, equivalence classes**

   (a) Check if the relation ρ is an equivalence relation over the alphabet $\Sigma = \{a, b\}$. The relation ρ: $(\forall u, v \in \Sigma^*)\ u \,\rho\, v$ if and only if u and v have a common prefix of the length of 2 or 3. Discuss all properties of an equivalence relation.

   (b) Find equivalence classes for the relation induced by the language of binary even numbers without useless zeros plus $\{0\}$.
   $\Sigma = \{0, 1\}$ $L = \{0, 10, 110, 10010, ...\}$

2. **Regular languages**

   (a) Prove using the pumping lemma that the following language is *not* regular :
   $\Sigma = \{a, b, c\}, L = \{a^{n-1}b^n c^{n-1}, n \geq 2\}$.

   (b) Prove using the Myhill-Nerode lemma whether the following language is regular or not.
   $\Sigma = \{a, b\}, L = \{a^n b^{3n}, n \in \mathbb{N} \wedge n \in [1, 20]\}$.

3. **Context–free grammar**

   (a) Transform the grammar into the Chomsky Normal Form.
   $G = \{V = \{A, B, C, S\}, T = \{0, 1\}, P, S\}$
   $P:\ S \to ACS \mid AB1$
   $A \to 0S \mid 0$
   $B \to 0BS \mid 1$
   $C \to AC \mid CAB \mid \epsilon$

   (b) Transform the grammar into the Greibach Normal Form.
   $G = \{V = \{A, B, C, S\}, T = \{\alpha, \beta, \gamma\}, P, S\}$
   $P:\ S \to ABC \mid BCA$
   $A \to AC\beta \mid BB\alpha$
   $B \to \alpha$
   $C \to \gamma \mid A\gamma$

4. **Context-free languages**

   (a) Prove that the language L over the alphabet $\Sigma = \{a, b\}$ is not a context–free

   $$L = \{a^i b^{i^2} : i \in N_+\}$$

   (b) Check using CYK algorithm whether the word **baccb** belongs to the language generated by the context-free grammar.
   $G = (\{A, B, C, S\}, \{a, b\}, P, S)$
   P**:** $S \to AB$
   $A \to BA|CS|a$
   $B \to SA|CB|b$
   $C \to AA|c$

5. **Turing Machines and automata**

   (a) Design a deterministic push–down automation that accepts the language $L = \{a^n b^{n+1} : n \geq 1\}$ over the alphabet $\Sigma = \{a, b\}$.

   (b) Construct a Turing machine in any model that accepts the language:
   $L = \{a^i b a^i : i \geq 1\}$.

6. **Finite automata construction**

   (a) Using the Myhilla-Nerode theorem construct a deterministic finite automation that accepts the language over the alphabet $\Sigma = \{0, 1\}$ that describes binary numbers without useless zeros, which can be divided by 8. The 0 belongs to the language.

   (b) Using quotient of languages design a deterministic finite automaton for the expression $(1 + \epsilon)(01)^*(0 + 1)(11)^*$

7. **Nondeterministic finite automata**

   (a) Transform a non–deterministic finite automaton with ϵ-moves into a non–deterministic finite automaton without ϵ-moves.

$$M = (Q = \{q_0, q_1, q_2, q_3, q_4\}, \Sigma = \{0, 1\}, \rho, q_0, F = \{q_3\})$$

ρ	0	1	ϵ
q_0	$\{q_2\}$	$\{q_1\}$	$\emptyset$
q_1	$\emptyset$	$\{q_2\}$	$\{q_0\}$
q_2	$\emptyset$	$\{q_4\}$	$\{q_3\}$
$\vec{q_3}$	$\{q_1\}$	$\{q_1\}$	$\emptyset$
q_4	$\{q_4\}$	$\emptyset$	$\{q_2\}$

   (b) Transform a nondeterministic finite automaton without ϵ-moves into a deterministic form.

$$M = (Q = \{q_0, q_1, q_2, q_3, q_4\}, \Sigma = \{0, 1\}, \rho, q_0, F = \{q_3\})$$

ρ	0	1
q_0	$\{q_2, q_3\}$	$\{q_0, q_1\}$
q_1	$\{q_2, q_3\}$	$\{q_0, q_1, q_2, q_3\}$
q_2	$\{q_0, q_1\}$	$\{q_2, q_3, q_4\}$
$\vec{q_3}$	$\{q_0, q_1\}$	$\{q_2, q_3, q_4\}$
q_4	$\{q_2, q_3, q_4\}$	$\emptyset$