Algorytmy rojowe w optymalizacji portfela inwestycyjnego

Swarm intelligence for portfolio optimization

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Investment portfolio

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Example

How to invest 100'000 USD into Apple and Gold CFD?

Portofolio optimizaton problem

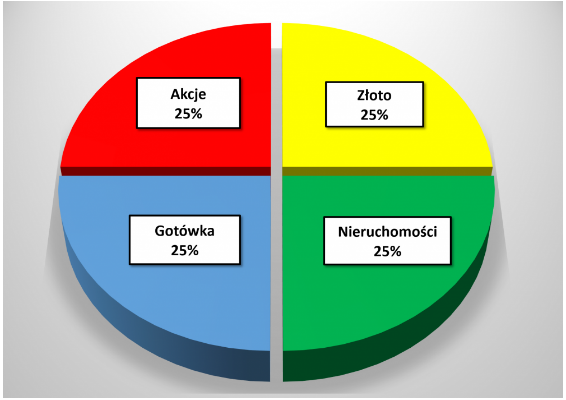


Figure 1: Mark Faber's portfolio[8]

Portofolio optimizaton problem

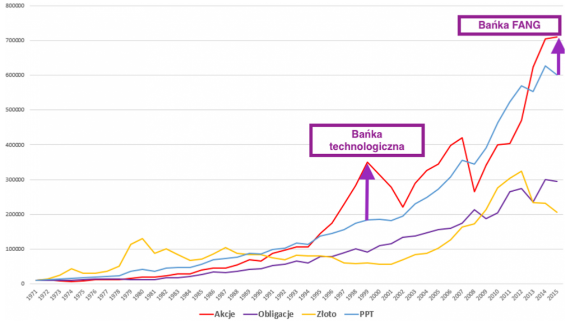


Figure 2: Mark Faber's portfolio performance[8]

Portofolio optimizaton problem

Questions:

- Do we want to invest full amount?
- Are short positions available?
- What is our goal?
  - Maximize profit?
  - Minimize risk?
  - Both?
- Do we accept full investments into one stock?
- How to assume (predict) future returns?

Portofolio optimizaton problem

Portfolio (investment portfolio) is a collection of investments / asset distribution

Portfolio optimization is the process of selecting the best portfolio according to assumed measure and constraints.

Modern Portfolio Theory

Modern portfolio theory[13]

Coca Cola yearly returns

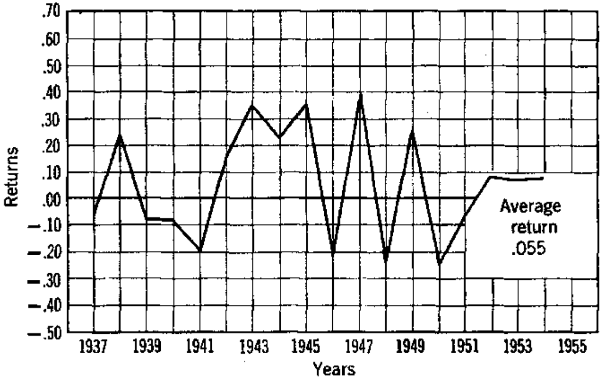


Figure 1f. Returns on security 6, Coca-Cola, Common.

Modern portfolio theory[13]

Santa Fe yearly returns

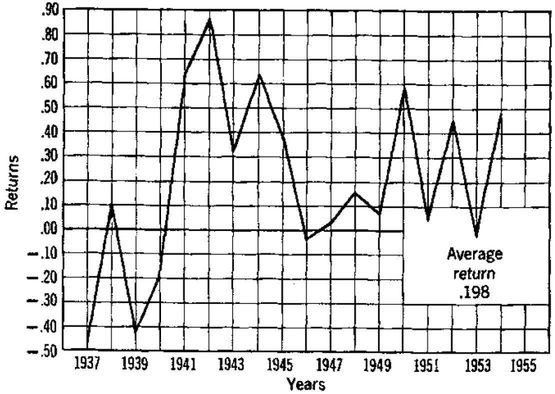


Figure 1e. Returns on security 5, Atchison, Topeka & Santa Fe.

Modern portfolio theory[13]

80% CocaCola + 20% Santa Fe yearly returns

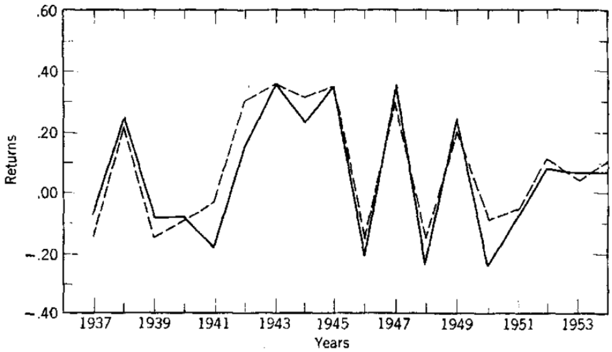


Figure 2. Returns on Coca-Cola Co. Common Stock and on a mixture of two securities (solid curve, Coca-Cola; dash curve, 80% C.C. + 20% A. T. & Sfe).

Modern portfolio theory

Definition[11]

- m risky assets: $i = 1, 2, \ldots, m$
- Single-Period Returns: $m-$variate random vector
$$\mathbf{R} = [R_1, R_2, \ldots, R_m]'$$
- Mean and Variance/Covariance of Returns:
$$E[\mathbf{R}] = \boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix}, \; Cov[\mathbf{R}] = \boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_{1,1} & \cdots & \Sigma_{1,m} \\ \vdots & \ddots & \vdots \\ \Sigma_{m,1} & \cdots & \Sigma_{m,m} \end{bmatrix}$$
- Portfolio: $m-$vector of weights indicating the fraction of portfolio wealth held in each asset
$$\mathbf{w} = (w_1, \ldots, w_m) : \sum_{i=1}^{m} w_i = 1.$$
- Portfolio Return: $R_{\mathbf{w}} = \mathbf{w}'\mathbf{R} = \sum_{i=1}^{m} w_i R_i$ a r.v. with
$$\begin{aligned} \alpha_{\mathbf{w}} &= E[R_{\mathbf{w}}] &= \mathbf{w}'\boldsymbol{\alpha} \\ \sigma^2_{\mathbf{w}} &= var[R_{\mathbf{w}}] &= \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} \end{aligned}$$

Modern portfolio theory

Single-objective approaches

1. Risk minimization with target return[11]:

Minimize: $\frac{1}{2}\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w}$
Subject to: $\mathbf{w}'\boldsymbol{\alpha} = \alpha_0$
$\mathbf{w}'\mathbf{1}_m = 1$

2. Expected return maximization with target variance (risk)[11]:

Maximize: $E(R_{\mathbf{w}}) = \mathbf{w}'\boldsymbol{\alpha}$
Subject to: $\mathbf{w}'\boldsymbol{\Sigma}\mathbf{w} = \sigma_0^2$
$\mathbf{w}'\mathbf{1}_m = 1$

Modern portfolio theory

Single-objective approaches

3. Arrow-Pratt risk aversion ($\lambda \geq 0$)[11]:

Maximize: $\left[E(R_{\mathbf{w}}) - \frac{1}{2}\lambda var(R_{\mathbf{w}})\right] = \mathbf{w}'\alpha - \frac{1}{2}\lambda\mathbf{w}'\mathbf{\Sigma}\mathbf{w}$

Subject to: $\mathbf{w}'\mathbf{1}_m = 1$

4. Other weighing methods
5. Sharpe Ratio model[14]

$$SR = \frac{R_p - R_f}{StdDev(p)}$$

6. ...

Modern portfolio theory

Alternative risk measures

Alternative risk measures:

- Variance with skewness (VwS)
- Value-at-Risk (VaR)
- Conditional Value-at-Risk (CVaR) / Expected shortfall (ES)
- Mean-Absolute Deviation (MAD)
- Minimax (MM)
- Maximum Drawdown (MDD)

Modern portfolio theory

Constraints

Unconstrained (UC) [2]:	- Ensure the investment of all funds - Do not allow short sales
Boundary Constraints (BC), also known as buy-in threshold [9]:	- Limit the minimum and maximum investment
Cardinality Constraints (CC) [10]:	- Restrict the number of assets in the portfolio
Transaction Costs (TC) [11]:	- Deduct transaction costs from the outcome
Transaction Lots (TL) [7]:	- Round the minimum and maximum number of transaction lots to nearest integer value

Figure 3: Constraints for realistic portfolio management. [6]

Modern portfolio theory

Constraints

Additional constraints (targets) examples:

- Risk-free rate
- Minimum and maximum group weights
- Turnover (reduce rebalancing cost)
- Concentration / concentration penalizing
- Tax

Modern portfolio theory

Multi-objective approaches

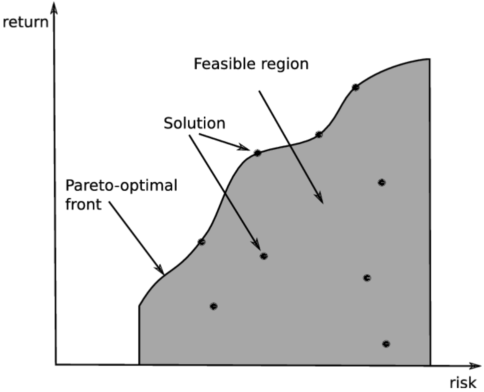


Figure 4: Pareto optimal front[5]

Modern portfolio theory

Multi-objective approaches

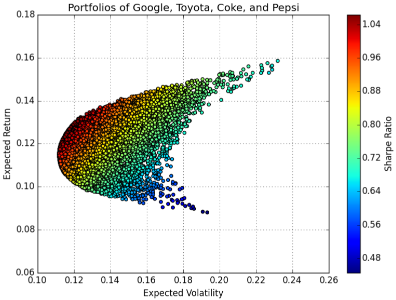


Figure 5: Monte Carlo simulation for portfolios of differing weights of Google, Toyota, Coke, and Pepsi stock[10]

Modern portfolio theory

Multi-objective approaches

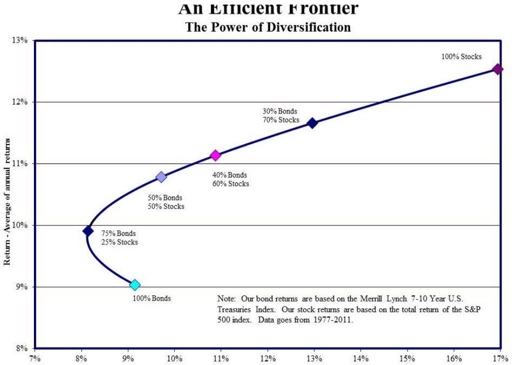


Figure 6: Efficient frontier[7]

Modern portfolio theory

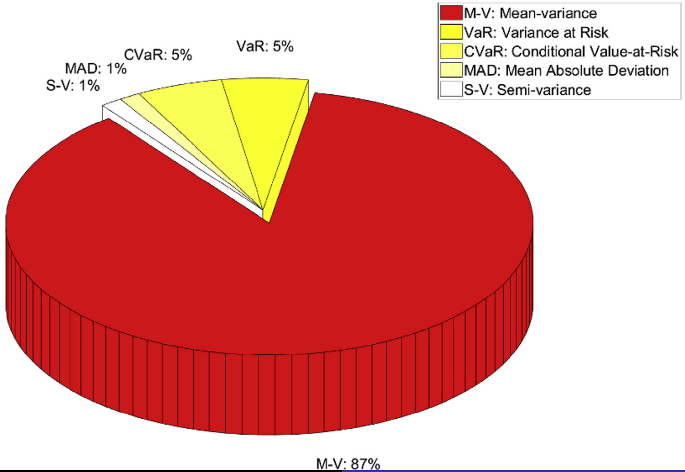
Models for portfolio optimziation

Model	Proposed By	Structure	Year
Mean-variance (M-V)	Markowitz [2]	Quadratic	1952
Variance with skewness (VwS)	Samuelson [4]	Quadratic	1958
Semi-variance (S-V)	Markowitz [3]	Quadratic	1959
Mean- Absolute Deviation (MAD)	Konno and Yamazaki [7]	Linear	1991
Value-at-Risk (VaR)	Jorion [5]	Linear	1997
Minimax (MM)	Young [8]	Linear	1998
Conditional Value-at-Risk (CVaR)	Rockafellar and Uryasev [6]	Linear	2000

Figure 7: Models for portfolio optimization. [6]

Modern portfolio theory

Models for portfolio optimziation



Modern portfolio theory disadvantages

Disadvantages:

- Risk expectation inaccuracy
- Vulnerability to stress (e.g. COVID-19)
- Weights instability

Modern portfolio theory extensions

Extensions

Extensions of MPT:

- Custom expected returns
- Non-normal distributions
- Fat-tail distributions
- Copula functions
- Black–Litterman model [2] (custom expected returns + confidence levels)

Portfolio optimization

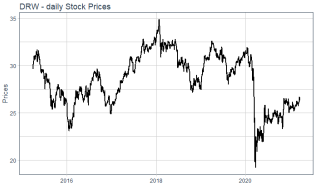
Portfolio optimization

Process

1. Gather historical prices
2. Take care of currency risk
3. Calculate returns
   - Daily (missing data, interpolation)
   - Weekly, monthly, etc.
4. Define measures (objective function)
   - Single-objective
   - Multi-objective
5. Define constraints
6. Try different portfolios and calculate objective function
   - Direct calculation
   - Simulation
7. Choose best one

Portfolio optimization

Practical example



Portfolio optimization

Practical example

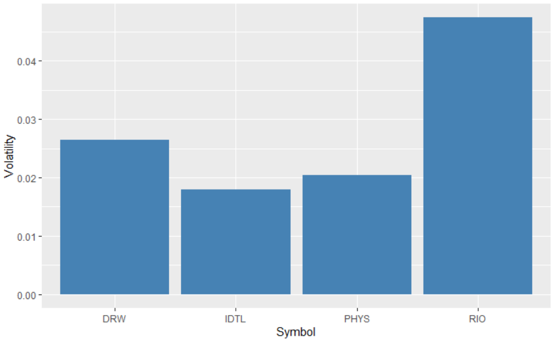


Figure 10: Stock volatility

Portfolio optimization

Practical example

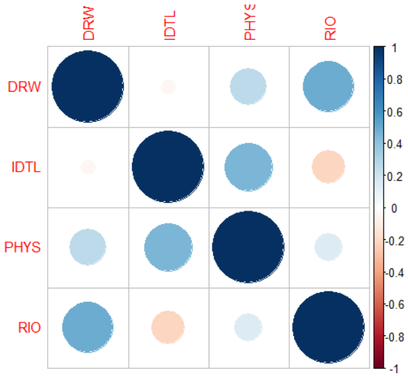


Figure 11: Correlations

Portfolio optimization

Practical example

Portfolio characteristic:

- Returns: simple, weekly
- Objective function: $\min \sigma^2$
- Full investment
- No short positions
- Weights ≤ 0.4

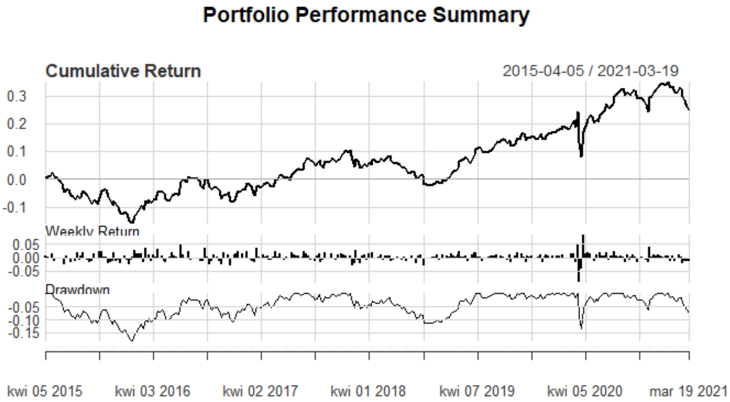
Weights:

- DRW - 0.2352
- IDTL - 0.4040
- PHYS - 0.3103
- RIO - 0.0506

Performance: $\sigma^2 = 0.1200833$

Portfolio optimization

Practical example



Portfolio optimization

Challenges

Challenges:

- Return expectations
- Risk measures
- Constraints definition
- **Custom models (objective function)**
- **Computation complexity**

Portfolio optimization

Solving techniques

Solving techniques:

- Random / Monte Carlo techniques
- Linear programming - efficient for unconstrained portfolio
- Quadratic programming - efficient for unconstrained portfolio
- Mixed-integer quadratic programming - for cardinality constraints
- Heuristic/metaheuristic algorithms:
  - Evolutionary Algorithms (EA)
  - **Swarm Intelligence (SI)**

Swarm-based algorithms adopted for Portfolio Optimization

Swarm-based algoritms adopted for Portfolio Optimization

Algorithm	Proposed by	Proposed in	Inspired by	Original Application area	Exploitation/Intensification mechanism	Exploration/Diversification mechanism
PSO	Kennedy and Eberhart [14]	1995	The swarming behavior such as fish and bird schooling in nature	Continuous domain	Less velocity rates	Greater velocity rates
ACO	Dorigo et al. [15]	1996	The foraging behavior of social ants	Discrete domain	Construction of solutions according to heuristic information	Probabilistic selection procedure utilizing pheromone information
BFO	Passino [16]	2002	The bacterial swarming and social foraging behaviors	Continuous domain	Chemotaxis and reproduction steps	Elimination-dispersal step
ABC	Karaboga [17]	2005	The foraging behavior of social bees	Continuous domain	Neighborhood search carried by employed and onlooker bees	Random search of scout bees
CSO	Chu et al. [18]	2006	The behaviors of cats in seeking and tracing modes	Continuous domain	Tracing mode	Seeking mode
FA	Yang [44]	2009	The flashing patterns and behavior of tropical fireflies	Continuous domain	Firefly movement according to attractiveness	Random move of the best firefly
IWO	Karimkashi and Kishk [20]	2010	the phenomenon of colonization of invasive weeds in nature	Continuous domain	Less standard deviation	high standard deviation
BA	Yang [21]	2010	The echolocation behavior of micro bats	Continuous domain	intensive local search controlled by the loudness and pulse rate	frequency tuning
FWA	Tan and Zhu [22]	2010	observing fireworks explosion	Continuous domain	small explosion amplitude	large explosion amplitude

Figure 12: Swarm based algorithms adopted for PO [6]

Swarm-based algoritms adopted for Portfolio Optimization

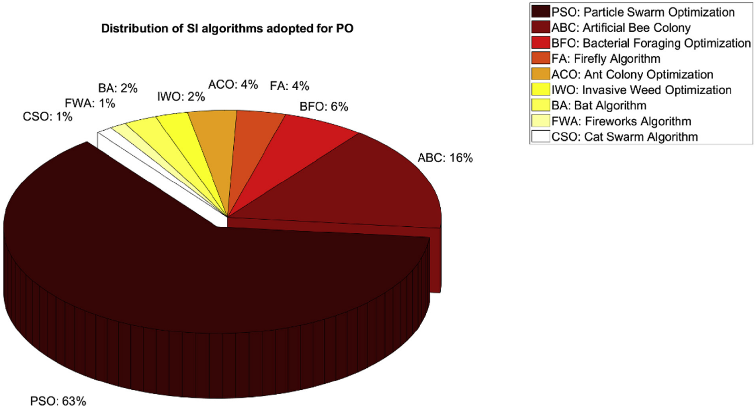


Figure 13: Distribution of SI algorithms adopted for PO. [6]

Swarm-based algoritms objectives

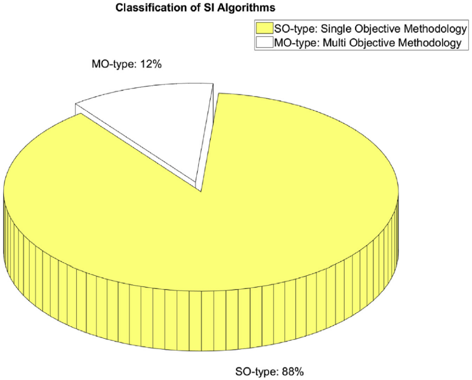


Figure 14: Classification of SI Algorithms according to objective handling methodology [6]

PSO principle

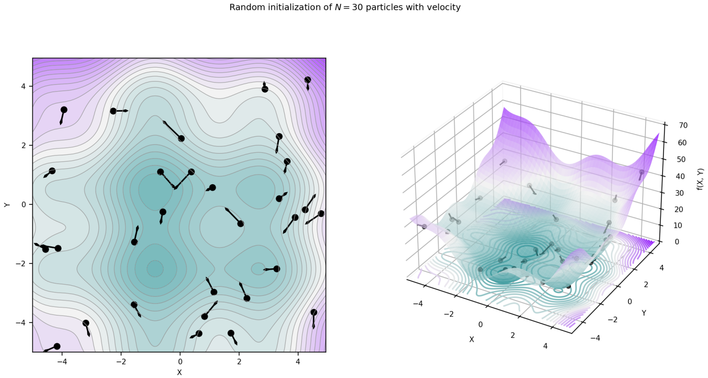


Figure 15: PSO visualisation[9]

PSO principle[3]

$$\begin{cases} v_{i,d}(t+1) = w v_{i,d}(t) + \\ \quad U(0,c)\left(p_i(t) - x_i(t)\right) + U(0,c)\left(l_i(t) - x_i(t)\right) \end{cases}$$

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

f is the fitness function defined on E. We suppose here that we are looking for its minimum

t is the number of the current time step

$x_i(t)$ is the position of the particle i at time t. It has D coordinates.

$v_i(t)$ is the velocity at time t. It has D components.

$p_i(t)$ is the previous best position, at time t. It has D coordinates.

$l_i(t)$ is the best previous best position found in the neighbourhood. It has D coordinates.

PSO principle

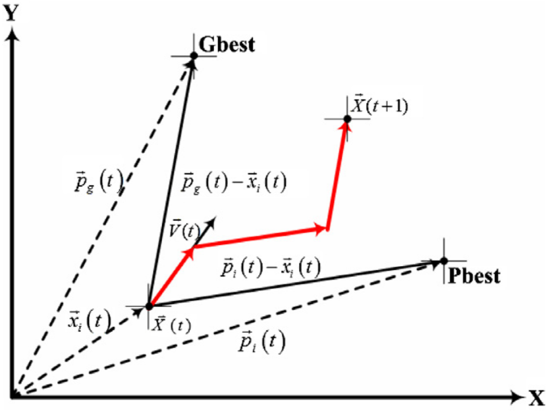


Figure 16: Construction of the next position[14]

PSO principle

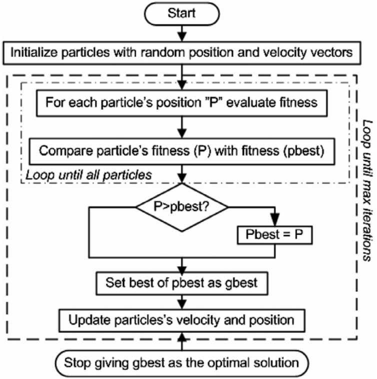


Figure 17: The flowchart depicting the general algorithm of PSO [14]

Constraint handling

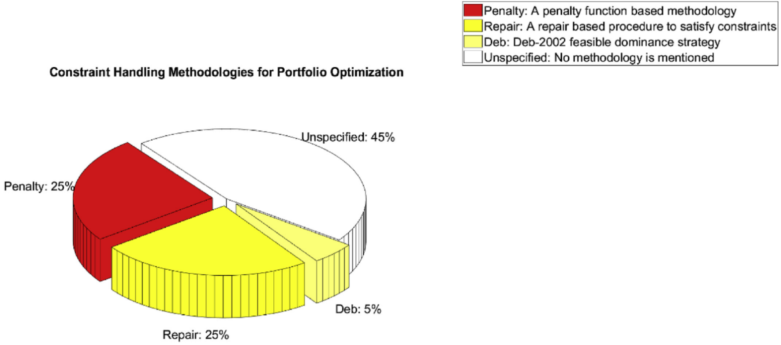


Figure 18: Distribution of constraint handling methodologies employed in SI algorithms adopted for PO [6]

Constraint handling[6]

- **Repair procedures** - designed to rearrange proportions (weights) of securities to satisfy all constraints
  - Reference (feasible) population [1]
  - Gradient-based techniques [1]
- **Penalty functions** - added to original fitness functions for leading the algorithm to feasible solutions in the end
- **Deb et al.[4]'s strategy**
  1. Pick 2 solutions from the population
  2. Choose the solution with better objective function value
  3. Choose the feasible solution
  4. Choose the solution with smaller overall constraint violation

MOPSO principle

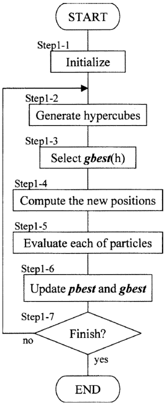


Figure 19: Framework of MOPSO [12]

MOPSO principle

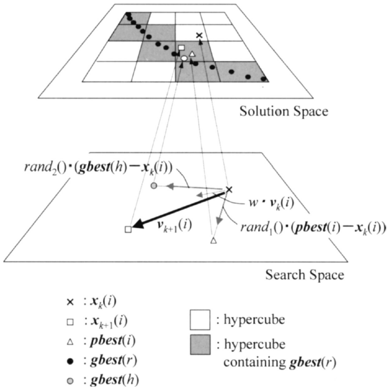


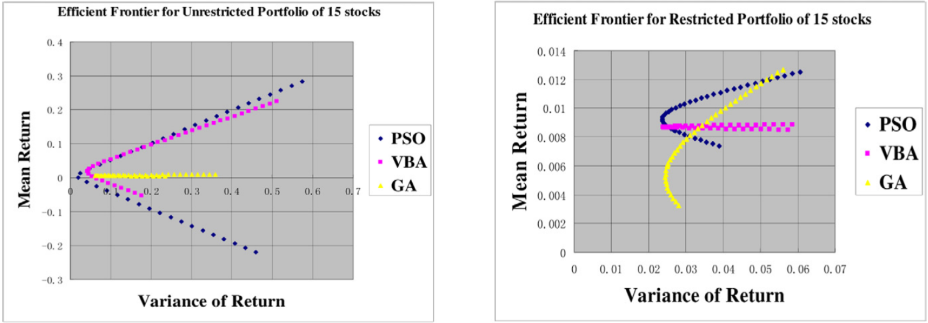
Figure 20: Concept of search point in search space and hypercubes in search space [12]

PSO in Sharp Ratio optimization

Approach	Item	Portfolio					
		8 Stocks		15 Stocks		49 Stocks	
		Unrestricted (%)	Restricted (%)	Unrestricted (%)	Restricted (%)	Unrestricted (%)	Restricted (%)
PSO solver:	ER	1.14	0.72	15.43	0.84	152.19	0.95
	SD	4.22	2.90	30.91	2.76	111.04	4.30
	Sharpe Ratio	19.84	17.83	48.96	26.73	136.78	15.06
GA solver:	ER	0.60	0.53	0.53	0.57	1.86	0.22
	SD	4.17	2.63	6.71	2.47	11.53	2.73
	Sharpe Ratio	7.24	12.42	3.48	18.96	13.56	−2.99
VBA solver:	ER	1.03	0.76	1.81	0.87	3.02	0.91
	SD	3.78	3.17	4.24	2.94	2.89	3.92
	Sharpe Ratio	19.31	17.55	35.70	26.31	94.02	12.90
Risk free		0.03	0.02	0.03	0.01	0.03	0.04

Figure 21: Six portfolios' results of PSO solver, GA solver and Excel solver [14]

(MO)PSO efficient frontier



c 15 stocks Unrestricted portfolio

d 15 stocks Restricted portfolio

Figure 22: Efficient frontier obtained during PSO portfolio optimization (SR) [14]

Summary

- Portfolio optimization is a multidisciplinary problem
- Portfolio optimization is a complex problem
- There is no one universal approach
- Swarm intelligence is useful in portfolio optimization

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