DM 10 Planar Graphs

10.1 A plane graph is *self-dual* if it is isomorphic to its dual ($G \equiv G^*$).

a) Show that if a graph G is self-dual, then $e(G) = 2|G| - 2$,

b) For each $n \geq 4$ find a self-dual graph on n vertices.

10.2 Show that if $|G| \geq 11$, then G or $\overline{G}$ is nonplanar.

10.3 Let $t(G) = \min\{k \in \mathbb{N}$: there exist k planar graphs $H_1, H_2, \ldots, H_k$ such that $E(G) = E(H_1) \cup E(H_2) \cup \ldots \cup E(H_k)\}$. Show, that $t(G) \geq \left\lceil \frac{e(G)}{3|G|-6} \right\rceil$.

10.4 Find all nonplanar graphs with 6 vertices.

10.5 Are given graphs planar? a) Petersen graph, b) C_9^2, c) C_8^2, d) $C_5 \times C_5$, e) graphs on the blackboard.

10.6 Prove or disprove: there exists a plan graph with 5 regions such that every two regions share an edge.

10.7 Prove or disprove: there exists a plane graph which is 5-connected.

10.8 Prove or disprove: there exists a 3-regular, bipartite, plan graph.

10.9 Show that if G is a simple bipartite, planar grap, then G^* is eulerian.

10.10 Prove (without 4-colour Theorem) that every trianglefree planar graph is 4-colourable.

10.11 Prove 4-colour Theorem for pour: Let V be the any set of identical coins on a table, there is an edge between two coins if they touch each other. Such graph is 4-colourable.