DM 2 Combinatorial identities, Partitions

2.1 How many possible results are there if we through n dice? Dice are a) different b) identical.

2.2 How many ways are there to choose committee consisting of 5 people of 10 nations, if there is more than one nation in the committee? Two people of the same nation are identical.

2.3 How many way are there to distribute k zlotys to n people if every one is to get a) any number of zlotys, b) at least one zloty?

2.4 Show by combinatorial arguments:

$a)\ \sum_{k=0}^{n}\binom{n}{k} = 2^n,$

$b)\ \binom{n}{k} = \binom{n}{n-k},$

$c)\ \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1},$

$d)\ \sum_{k=0}^{n}(-1)^k\binom{n}{k} = 0,\ n \neq 0$

$e)\ \binom{n+m}{k} = \sum_{i=0}^{k}\binom{n}{i}\binom{m}{k-i},$

$f)\ \binom{2n}{n} = \sum_{i=0}^{n}\binom{n}{i}^2,$

$g)\ \binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k},$

$h)\ \binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1},$

$i)\ \sum_{i=0}^{n}\binom{n}{i}i = n2^{n-1},$

$j)\ \sum_{k=1}^{n}k^2\binom{n}{k} = n(n+1)2^{n-2},$

$k)\ \sum_{j=0}^{n}\binom{n}{j}\binom{n-j}{k-j} = \binom{n}{k}2^k,$

$l)^*\ m^n = \sum_{k=0}^{n}\binom{n}{k}(m-1)^{n-k}.$

$m)^*\ \sum_{r=0}^{n}\binom{r}{k} = \binom{n+1}{k+1},$

$n)^*\ \sum_{k=0}^{n}\binom{r+k}{k} = \binom{r+n+1}{n},$

2.5 Evaluate a) $S(5,3)$, b) $S(7,5)$, c) $S(n,n-1)$, d) $S(n,2)$, where $n \geqslant 2$.

2.6 List all partitions of a 5-element set.

2.7 List all partitions of a 6-element set into 3 blocks.

2.8 Evaluate a) B_5, b) B_7.

2.9 Evaluate a) $P(9,5)$, b) $P(11,4)$, c) $P(13,8)$, d) $P(n,2)$, where $n \geqslant 2$.

2.10 Show that the number of partitions of an integer $n > 0$ into pairwise different components is equal to the number of partitions of n into odd components.

2.11 Show that $P(n,3)$ is equal to the number of partitions of $2n$ into 3 components such that each of them is smaller than n.

2.12 Show that $P(2n,n)$ is equal to the number of all partitions of n.

2.13 Show that the number of partitions of an integer n into no more than k components is equal to the number of partitions of $n + \frac{k(k+1)}{2}$ into k pairwise different components.

2.14 Show that the number of partitions of an integer n into even components is equal to the number of partitions of n such the number of every component is even.

2.15 Show that the number of partitions of an integer n into odd components is equal to the number of partitions of n such the number of every component (except the biggest one) is even.