

DM 3 The Inclusion-Exclusion Formula.

3.1 How many integers from 1 to 1000 are divisible by none of:

- a) 2, 6, 13; b) 3, 7, 11; c) 6, 63, 144.

3.2 Each of class of 50 students reads at least one of mathematics and physics, 30 read mathematics and 27 read both. How many read physics?

3.3 How many permutations are there of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, such that the first digit is greater than 2 and the last one smaller than 7?

3.4 How many permutations are there of the digits 1, 2, 3, 4, 5, 6, 7, 8 in which none of the patterns 12, 34, 56, 78 appears?

3.5 a) How many ways are there of placing 5 non-takin rooks on 5×5 board?

b) How many ways if none lies on the main diagonal?

c) How man ways if exactly one lies on the main diagonal?

3.6 Given $2n$ letters, two of each of n types. How many arrangements of these letters are there with no pair of consecutive letters the same?

3.7 There are five letters to different people and five addressed envelopes. How many ways are there to put letters into envelopes (one into each) in such a way that nobody gets his letter?

3.8 There is a caravan with 9 camels. In how many ways can we rearrange the order of camels in such a way that no camel goes after the camel it went after before the rearrangement?

3.9 There was a party for 10 people. After the party everyone tries to put two shoes on. How many ways are there in which they can do it, such that nobody takes any of his shoes. We assume that anybody can try with two left or two right shoes or one left and one right.

3.10 There are n pairs of shoes. In how many ways can we put them in a line in such a way that shoes of the same pair are not next to each other.

3.11 How many integer solutions are there of the equation $x_1 + \dots + x_6 = 30$ if:

a) $0 \leq x_i \leq 10, i \in \{1, \dots, 6\}$,

b) $-10 \leq x_i \leq 20, i \in \{1, \dots, 6\}$,

c) $0 \leq x_i, i \in \{1, \dots, 6\}, x_1 \leq 5, x_2 \leq 10, x_3 \leq 15, x_4 \leq 20$.

3.12 How many sequences of length $n, n \geq 3$ consisting of digits 0, 1, ..., 9 are there such that every digit appears at least once?

3.13 How many $n \times n$ 0-1 matrices are there with at least one row consisting of zeros only?

3.14 In how many ways n marriage couples can sit at a round table in such a way that no husband and wife sit next to each other?