DM6 Notation $O()$

$f(n) = O(g(n))$ iff
$\exists n_0 \in \mathbb{N}\ \exists C \in \mathbb{R}\ \forall n \geq n_0 \quad f(n) \leq C \cdot g(n)$

$f(n) = \Omega(g(n)$ iff $g(n) = O(f(n))$

$f(n) = \Theta(g(n)$ iff
$f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$

6.1 Prove by definition
a) $\sqrt{2n+4} = O(n)$
b) $n^2 = O(2^n)$
c) $\sqrt{\sqrt{2n^2+4}+n} = O(n)$
d) $(n+a)^2 = \Theta(n^2)$ for any real number.

6.2 Find the smallest k such that $f(n) = O(n^k)$, where :
a) $f(n) = 5n^4 - 3n^3 + 2n^2 + 4n + 2$
b) $f(n) = (n+1)^2(3n^2 + 2n)$
c) $f(n) = \sqrt{n+100}$
d) $f(n) = \sqrt{n^2+3n}$
e) $f(n) = (1.5)^n$
f) $f(n) = \log_2 n$
g) $f(n) = n^2 \log_2 n$
h) $f(n) = \sqrt{n^5 + 3n^2 + 150} + n$
i) $f(n) = \frac{\sqrt{n^2+5n^3-6n+10}+n}{n+1}$

6.3 Show that:
a) $n! = O(n^n)$,
b) $\lg_2(n!) = O(n \lg_2 n)$,
c) $4^{\lg_2 n} = \Omega(2^{\lg_2 n})$,
d) $(n+1)! = \Omega(n!)$.

6.4 Insert either Θ or Ω or O. Explain your answer.
a) $2^{n+1} = \dots(2^n)$
b) $2^{2n} = \dots(2^n)$
c) $(2n)! = \dots(n!)$
d) $(\sqrt{n}+1)^4 = \dots(n^2)$
e) $n + 3\log_2 n = \dots(\log_2 n)$
f) $\sqrt{n}(n+3) = \dots(n \log_2 n)$
g) $\sqrt{n^2+1} + 3\log_2 n = \dots(n \log_2 n)$
h) $\sqrt{3n^3+2n^2+1} + n^2\sqrt{\log_2 n} = \dots(n^2)$
i) $\ln(\ln n) = \dots((\ln n)^{\ln n})$
j) $2^{\lg_2 n} = \dots(2^{\ln n})$
k) $\ln(n^{\ln n}) = \dots((\ln n)^{\ln n})$
l)* $2^{2^n} = \dots(n^n)$,
m)* $\ln(n^n) = \dots((\ln(n))^n)$,
o)* $\lg_2(n+1) = \dots(\ln(n^2))$,
p)* $5^{lg_2(n)} = \dots 2n+3$,
q)* $n^{\sqrt{n}} = \dots(\sqrt{n})^n$,
r)* $n^{\ln(c)} = \dots c^{\ln(n)}$, where c is some constant.