DM 8 Euler tours, Hamiltonian cycles

7.25 Find a graph G such, that:
a) $\delta(G) = n - 3$ and $\kappa(G) < \delta(G)$,
b) $\delta(G) = \lfloor \frac{n}{2} \rfloor - 1$ and $\kappa'(G) < \delta(G)$.

7.26 Show, that if $\delta(G) \geq \frac{n+k-2}{2}$ then $\kappa(G) \geq k$.

7.27 Prove that if G is bipartite and k-regular ($k \geq 2$) then G has no bridge.

7.28. Show, that for evety 3-regular graph G $\kappa(G) = \kappa'(G)$.

7.29. For any natural numbers $a \leq b \leq c$ find a graph G such, that $\kappa(G) = a$, $\kappa'(G) = b$, $\delta(G) = c$.

8.1 Does there exist a graph with
a) an odd number of vertices and an odd number of edges,
b) an odd number of vertices and an even number of edges,
c) an even number of vertices and an odd number of edges,
d) an even number of vertices and an even number of edges
which has a) an Euler tour, b) a Hamilton cycle ?
(If YES, then draw an example. If NO, then explain why there is no such graph.)

8.2 Find (if there exists, otherwise, explain why there is no such graph) an example of a graph which is
a) hamiltonian and eulerian,
b) hamiltonian and noneulerian,
c) nonhamiltonian and eulerian,
d) nonhamiltonian and noneulerian.

8.3 Show that if a graph G has $2k > 0$ vertices of odd degree, then there are k edge-disjoint trails $Q_1, Q_2, \ldots, Q_k$ in G such that $E(G) = E(Q_1) \cup E(Q_2) \cup \ldots \cup E(Q_k)$.

8.4 Show that if there exists a vertex v in a graph G such that $deg_G v > \frac{|G|-1}{2}$ and $G \setminus v$ is hamiltonian, then G has a Hamilton cycle.

8.5 Prove that if either G is not 2-connected or G is bipartite with bipartition (X, Y), where $|X| \neq |Y|$ then G is not hamiltonian.

8.6 Show an example of a tree T such that T^2 has no Hamilton cycle ($T^2 = (V(T), \{(u, v) : u, v \in V(T)$ and $dist_T(u, v) \leq 2\})$).

8.7 Show that if G has a Hamilton path then, for every proper subset S of $V(G)$, $\omega(G \setminus S) \leq |S| + 1$.

8.8 Show that if G is a graph, $|G| \geq 5$, such that for each pair of vertices u, v there is an $u - v$ Hamilton path in G, then $\kappa(G) \geq 3$.

8.9 Show that 6-regular graph on 11 vertices has a Hamiltonian cycle.