DM 9 COLOURINGS

EDGE COLOURINGS

9.1 Find optimal edge colouring of $K_{n,m}$, the Petersen graph.

9.2 Prove, that if G is 3-regular hamiltonian graph then $\chi'(G) = 3$.

9.3 Let M and N be edge-disjoint matchings in G such that $|M| > |N|$. Prove that there exist edge-disjoint matchings M' and N' such that $|M'| = |M| - 1$, $|N'| = |N| + 1$ and $M \cup N = M' \cup N'$.

9.4 Show, that for any graph G and any $p > \Delta(G)$ there exists p disjoint matchings $M_1, M_2, \ldots, M_p$ in G such that $E(G) = M_1 \cup M_2 \cup \ldots \cup M_p$ and

$$\left\lfloor \frac{e(G)}{p} \right\rfloor \leq |M_i| \leq \left\lceil \frac{e(G)}{p} \right\rceil .$$

for $1 \leq i \leq p$

VERTEX COLOURINGS

9.5 What is the vertex chromatic number of the graph obtained from K_n by removing a) one edge, b) two edges (consider two cases), c) three edges that form a triangle.

9.6 Show that if G is a 3-regular graph with a Hamilton cycle then $\chi'(G) = 3$.

9.7 Prove that for each graph G:
$\chi(G)\chi(\overline{G}) \geq |G|$.

9.8 Let $(d_1, .., d_n)$ be degree sequence of graph G.
Show that $\chi(G) \leq \max_{i \in \{1,..,n\}} \min\{d_i + 1, i\}$.

9.9 Show that graph G can be properly coloured on
a) $\Delta(G) + 1$ colours,
b) $\Delta(G)$ colours if $\delta(G) < \Delta(G)$ and G is connected. Why assumption of connectedness is essential?

9.10 Show, that if graph G is colored with $\chi(G)$ vertices then for any two colors there is an edge with ends in these colors.