1. Assuming that $A$ is the matrix of a linear operator $F$ in $S$ find the matrix $B$ of $F$ in $R$:

(a) \[ A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad S = \{(1,1),(1,2)\}, \quad R = \{(1,0),(0,1)\} \]

(b) \[ A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad S = \{(1,1,1),(0,1,2),(1,0,1)\}, \quad R = \{(1,0,0),(0,1,0),(0,0,1)\} \]

(c) \[ A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 0 \\ 1 & 2 & 3 \end{bmatrix}, \quad S = \{(1,1,1),(0,1,2),(1,0,1)\}, \quad R = \{(1,0,0),(0,1,0),(0,0,1)\} \]

(d) \[ A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad S = \{(0,0,1),(0,1,0),(1,0,0)\}, \quad R = \{(1,1,0),(1,1,1),(0,1,1)\} \]

(e) \[ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix}, \quad S = \{(1,0,1,0),(0,0,1,1),(1,1,0,0),(1,0,1,1)\}, \quad R = \{(1,1,1,0),(1,1,0,1),(1,0,1,1),(0,1,1,1)\} \]

2. For each two matrices $A$ and $B$ from problem 1 find the change-of-basis matrix $P$ such that $A = P^{-1}BP$. Verify your solution by matrix multiplication.

3. For each operator from problem 1 determine if there exists a basis $T$ such that the matrix of the operator in $T$ is diagonal.

4. Find all eigenvalues and a basis of each eigenspace for the operator $F: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, $F(x,y,z) = (2x+y,y-z,2y+4z)$

5. For each of the following matrices find all eigenvectors and a basis for each eigenspace.

(a) \[ A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 3 & 3 \end{bmatrix} \]

(b) \[ B = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -1 & 1 & 4 \end{bmatrix} \]

(c) \[ C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]