TUTORIAL 6. LINEAR INDEPENDENT SETS. SPANNING SETS.

6.1. Decide whether the given set of vectors is linearly independent in the indicated vector space.
   a. \{ x^4 + x^3 + x^2 + x + 1, x^2 + x + 2, x \} in \( R[x] \) over \( R \),
   b. \{ (1,0,1), (1,1,0), (0,1,1) \} in \( Z_3^3 \) over \( Z_2 \),
   c. \{ (1,0,1), (1,1,0), (0,1,1) \} in \( R^3 \) over \( R \),
   d. \{ \sin x, \cos x, x \} in \( R^3 \) over \( R \),
   e. \{ \sin^2 x, \cos^2 x, \sin 2x, \cos 2x \} in \( R^4 \) over \( R \),
   f. \{ x_1, x_1 + x_2, x_1 + x_2 + x_3, \ldots, x_1 + \ldots + x_n \} if \{ x_1, x_2, x_3, \ldots, x_n \} is linearly independent, in any vector space \( V \),
   g. \{ x_1 + x_2, x_2 + x_3, \ldots, x_{n-1} + x_n, x_n + x_1 \} if \{ x_1, x_2, x_3, \ldots, x_n \} is linearly independent, in any vector space \( V \),
   h. \{ x_1 + v, x_2 + v, \ldots, x_n + v \} if \{ x_1, x_2, x_3, \ldots, x_n \} is linearly independent and \( v \) is any vector,
   i. \( S \), where \( S \subseteq T \) and \( T \) is linearly independent,
   j. \( T \), where \( T \subseteq S \) and \( T \) is linearly independent,
   k. \{ x_1, x_2, x_3, \ldots, x_n \}, where \( x_i \in \text{span}(S_i), x_i \neq \Theta \) for \( i = 1, 2, \ldots, n \), sets \( S_i \) are finite and pairwise disjoint and the set \( S_1 \cup S_2 \cup \ldots \cup S_n \) is linearly independent.

6.2. Find the dimension of each of the following vector spaces:
   a. \( R \) over \( Q \),
   b. \( C \) over \( R \),
   c. All polynomials with real coefficients, of degree \( \leq 7 \), with a root at 1, over \( R \),
   d. \( F^n \) over \( F \), where \( F \) is any field,
   e. All polynomials with real coefficients, of degree \( \leq 8 \), divisible by \( x^2 + 1 \), over \( R \),
   f. \( \text{Span}(\{ \sin^2 x, \cos^2 x, \sin 2x, \cos 2x \}) \) over \( R \),
   g. \( \text{Span}(\{ x_1, x_2, x_3, \ldots, x_n \}) \), where \( \{ x_1, x_2, x_3, \ldots, x_n \} \) is a basis for \( V \),
   h. \( \text{Span}(\{ x_n, x_{n-1}, x_{n-2}, \ldots, x_1 \}) \), where \( \{ x_1, x_2, x_3, \ldots, x_n \} \) is a basis for \( V \),
   i. \( \{ (x,y,z,t) \in R^4 \mid x+y+z+t=0 \} \),
   j. \( \{ (x,y,z,t) \in R^4 \mid 2x+y=z-t \} \),
   k. \( \text{span}(\{(1,2,1,0),(1,3,1,1),(0,1,2,-1),(-3,1,2,2)\}) \) over \( R \),
   l. \( 2^{(a,b,c)} \) over \( Z_2 \).