

## ItMM 0 Induction

Prove by Induction:

$$0.1 \quad 1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + n \cdot 2^n = 2 + (n-1)2^{n+1}$$

$$0.2 \quad \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1},$$

$$0.3 \quad 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6},$$

$$0.4 \quad 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{1}{4}(1+n)^2 n^2,$$

$$0.5 \quad 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \dots + n \cdot (n+1) = \frac{1}{3}n(n+1)(n+2),$$

$$0.6 \quad 2^3 + 4^3 + \dots + (2n)^3 = 2n^2(n+1)^2,$$

$$0.7 \quad 1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \dots + n \cdot n! = (n+1)! - 1,$$

$$0.8 \quad 5n \leq n^2 - 3 \text{ dla } n \geq 6,$$

$$0.9 \quad n^3 < 4^n,$$

$$0.10 \quad 3^n > n2^n,$$

$$0.11 \quad \text{Bernoulli inequality } (1+a)^n \geq 1+n \cdot a, \quad a > -1, \quad n \in \mathbb{N},$$

$$0.12 \quad 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} > \sqrt{n}, \quad n \geq 2,$$

$$0.13 \quad 8|5^n + 2 \cdot 3^{n-1} + 1,$$

$$0.14 \quad 11|2^{6n+1} + 3^{2n+2},$$

$$0.15 \quad 133|11^{n+2} + 12^{2n+1}$$

$$0.16 \quad 9|4^n + 24n - 1.$$

$$0.17 \quad \text{The sequence } a_n \text{ is given by } a_0 = 1 \quad a_n = \frac{a_{n-1}}{2a_{n-1}+1}. \text{ Find and prove a formula for } a_n.$$

0.18 Prove there is infinite number of prime numbers.

0.19 Consider  $n$  points, for every two of them there is an arrow from one to another. A center is a point from which we can get to any other point in two steps going in the direction on arrows. Prove that a center always exists.

0.20 We will prove inductively that all cats are in the same color. Take a group consisting one cat. All cats in this group are in the same color. Induction step: assume any group of  $k$  cats are in the same color. Consider group of  $k+1$  cats. Take one of them, by induction assumption they are in the same color. Put it back and take another one, the rest is again of the same color. So all  $k+1$  cats are in the same color. The prove is complete, what is wrong?