

## IMM 3 Unions and intersections of families of sets

3.1 Find  $\bigcap_{i \in \mathbb{N}} A_i$  for:

- a)  $A_i = (0; \frac{1}{i+1})$ ,      b)  $A_i = (0; \frac{1}{i+1}]$ ,      c)  $A_i = [0; \frac{1}{i+1}]$ ,      d)  $A_i = (i; \infty)$   
e)  $A_i = \{(x, y) \in \mathbb{R}^2 : x + y \leq \frac{1}{i+1}\}$       f)  $A_i = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0, y \leq x^i\}$ .

3.2 Find  $\bigcup_{i \in \mathbb{N}} A_i$  for:

- a)  $A_i = (i; \infty)$ ,      b)  $A_i = (\frac{1}{i+1}; 1 - \frac{1}{i+1}]$ ,      c)  $A_i = (-i; i)$ ,  
d)  $A_i = \{(x, y) \in \mathbb{R}^2 : x + y \leq i\}$ ,      e)  $A_i = \{(x, y) \in \mathbb{R}^2 : x + y \leq \frac{1}{i+1}\}$ ,  
f)  $A_i = \{(x, y) \in \mathbb{R}^2 : x, y \geq 0, y \leq x^i\}$ .

3.3 Find  $\bigcup_a \bigcap_b X_{a,b}$ ,  $\bigcap_a \bigcup_b X_{a,b}$ ,  $\bigcap_b \bigcup_a X_{a,b}$ ,  $\bigcup_b \bigcap_a X_{a,b}$  for:

- a)  $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : 0 < x \wedge 0 \leq y \wedge \frac{x}{a} + \frac{y}{b} \leq 1\}$ ,  $a, b \in \mathbb{R}$ ,  $a, b > 0$ ,  
b)  $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \leq a(x - b)\}$ ,  $a, b \in \mathbb{R}$ ,  
c)  $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \leq ax(x - b)\}$ ,  $a, b \in \mathbb{R}$ ,  $a, b > 0$ ,  
d)  $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \geq \frac{a}{b^2}x(x - 2b)\}$ ,  $a, b > 0$ ,  
e)  $X_{a,b} = \{x \in \mathbb{R} : a - \frac{1}{b} \leq x < a + \frac{1}{b}\}$ ,  $a \in \mathbb{Z}$ ,  $b \in \mathbb{N} - \{0\}$ ,  
f)  $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : 0 < y \leq ax^2 + b\}$ ,  $a, b \in \mathbb{R}$ ,  $a < 0$ ,  $b > 0$ ,  
g)  $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y = a(x - b)^3 + b\}$ ,  $a, b \in \mathbb{R}$ ,  $a, b > 0$ ,  
h)  $X_{a,b} = \{(x, y) \in \mathbb{R}^2 : y \geq e^{a(x-b)}\}$ ,  $a, b \in \mathbb{R}_+$ ,  $a, b > 0$ ,  
i\*)  $X_{a,b} = \{(x, y) \in \mathbb{R}^{+2} : ax^2 < y \leq \sqrt[3]{ab^2}\}$ ,  $a, b \in \mathbb{R}$ ,  $a, b > 0$ ,

3.4 Find  $\bigcup_{i \in \emptyset} A_i$  and  $\bigcap_{i \in \emptyset} A_i$ .

3.5 What relations of inclusions hold? Find counterexamples for false ones.

- a)  $\bigcup_{i \in I} A_i \cap \bigcup_{i \in I} B_i$  and  $\bigcup_{i \in I} (A_i \cap B_i)$   
b)  $\bigcap_{i \in I} A_i \cup \bigcap_{i \in I} B_i$  and  $\bigcap_{i \in I} (A_i \cup B_i)$   
c)  $\bigcup_{i \in I} (A_i - B_i)$  and  $\bigcup_{i \in I} A_i - \bigcap_{i \in I} B_i$   
d)  $\bigcap_{i \in I} (A_i \cup -B_i)$  and  $\bigcap_{i \in I} A_i \cup -\bigcup_{i \in I} B_i$

3.6 Prove:

- a)  $(\bigcup_{i \in I} A_i) \times (\bigcup_{j \in J} B_j) = \bigcup_{i \in I} \bigcup_{j \in J} (A_i \times B_j)$ ,  
b)  $(\bigcap_{i \in I} A_i) \times (\bigcap_{j \in J} B_j) = \bigcap_{i \in I} \bigcap_{j \in J} (A_i \times B_j)$ ,

3.7 Find infinite family  $\mathcal{X}$  of subsets of  $\mathbb{N}$  such, that  $\bigcap \mathcal{X} = \emptyset$  and  $\bigcap \mathcal{Y} \neq \emptyset$  for every proper subfamily  $\mathcal{Y}$  of the family  $\mathcal{X}$ .